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Nominalism and constructivism in seventeenth-century mathematical philosophy

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Abstract

This paper argues that the philosophical tradition of nominalism, as evident in the works of Pierre Gassendi, Thomas Hobbes, Isaac Barrow, and Isaac Newton, played an important role in the history of mathematics during the 17th century. I will argue that nominalist philosophy of mathematics offers new clarification of the development of a "constructivist" tradition in mathematical philosophy. This nominalist and constructivist tradition offered a way for contemporary mathematicians to discuss mathematical objects and magnitudes that did not assume these entities were real in a Platonic sense, and helped lay the groundwork for formalist and instrumentalist approaches in modern mathematics.

Résumé

Cet article soutient que la tradition philosophique du nominalisme, évidente dans les travaux de Pierre Gassendi, Thomas Hobbes, Isaac Barrow et Isaac Newton, a joué un rôle important dans l'histoire des mathématiques pendant le dix-septième siècle. L'argument princicipal est que la philosophie nominaliste des mathématiques est à la base du développement d'une tradition « constructiviste » en philosophie mathématique. Cette tradition nominaliste et constructiviste a permis aux mathématiciens contemporains de pouvoir discuter d'objets et quantités mathématiques sans présupposer leur réalité au sens Platonique du terme, et a contribué au developpement desétudes formalistes et instrumentalistes des mathématiques modernes.

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In the middle decades of the 20th century, historians of 16th- and 17th-century mathematical sciences tended to focus on the conceptual or metaphysical foundations of mathematical natural philosophy. This

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approach, exemplified in the work of scholars such as E.A. Burtt [1952], Alexandre Koyré [1978], and E.J. Dijksterhuis [1961], had a major role in defining the period of time now traditionally known as the "scientific revolution." While this conceptualist program has been eclipsed in more recent decades by studies of the social, institutional, and technical context of early modern mathematical practice, there remains a strong tradition in intellectual history of mathematics. Recent scholarship by Paolo Mancosu [1996], Douglas Jesseph [1999], Niccolo Guicciardini [1999], Michael Mahoney [1998], and Peter Dear [1995] has analyzed intellectual traditions in mathematical practice and shown that 17th-century mathematical philosophy developed out of a more complex and heterogeneous web of influences than was envisioned by what H. Floris Cohen calls the "Great Tradition" in 20th-century historiography [Cohen, 1994, Ch. 2].[1]

One liability, however, in the greater degree of specialization with which historians have approached the history of early science has been a fragmentation of the older, more unified narratives. This is particularly the case in the history of mathematics. While we now have a better understanding of the diversity of mathematical practices, methods, and traditions in the 16th through 18th centuries, there is a need to connect the history of mathematics with a broader view and with attention to its general philosophical context. This need is more than simply historiographical: early modern natural philosophers themselves did not separate mathematical and scientific pursuits from more general questions in philosophy, so understanding the philosophical basis of their beliefs gives important insight into the development of contemporary mathematical natural philosophy.

In this paper, I will contextualize a particular approach to mathematical philosophy in the 17th century within the framework of a broader epistemological and philosophical movement: the early modern resurgence of the medieval tradition of nominalism. Nominalism has been connected by scholars to important epistemological and scientific developments in the 17th and 18th centuries, particularly in relationship to theological and biological questions.[2] It has not, however, been widely promoted as a factor in early mathematical practice and philosophy.[3] This paper will argue, nonetheless, that nominalistic strategies provided 17th-century practitioners, including Pierre Gassendi and Thomas Hobbes, with an important set of arguments for describing the ontological content of mathematical entities and magnitudes in a way that departed from Platonic realist accounts without sacrificing reference to empirical experience. This nominalistic approach, I will argue further, was closely linked with the development of a constructivist philosophy of mathematics, which has been linked by a number of recent scholars to the geometric and analytic practices of mathematicians such as Isaac Barrow and Isaac Newton. More generally, I hope to demonstrate the close relationship between mathematical philosophy and wider philosophical and theological beliefs during the scientific revolution, and to reinforce the historiographical point that, at least during the early modern period, the history of mathematics was inextricably linked with contemporary currents in broader intellectual culture.

[1] Cohen uses this term to refer to the historiographic model established by Burtt, Koyré, Dijksterhuis, and Herbert Butterfield.

[2] On the relationship between nominalism and early modern thought, see Dupre [1993, especially Chs. 3, 5, and 7] and Funkenstein [1986]. For a discussion of the relationship between medieval and early modern theology and nominalism, see Osler [1994, pp. 115–116]. On nominalism and the history of biology, see Stamos [1996, pp. 127–144] and Mayr [1976, pp. 429–430].

[3] Jesseph [1999, pp. 205–211] takes up the subject of Thomas Hobbes's nominalism, and he also discusses nominalism in relation to George Berkeley's formalist mathematics [1993, pp. 118–120].

The importance of the constructivist tradition in early modern mathematical practice has been highlighted in the recent work of Peter Dear, who traces the development of a technical and conceptual approach to mathematical natural philosophy from the 16th-century Continental Jesuit authors through the English mathematicians in the late 17th century. The connection he identifies between these two poles is a tendency towards constructivism. Serving as a common link between authors as divided in time and circumstances as Christoph Clavius (1538–1612) and Isaac Newton, Dear argues, constructivism provided an epistemological strategy that offered natural philosophers a bridge between discrete individual experiences and generalized mathematical entities. As he explains,

> Newton's work drew directly from the tradition of mathematical sciences examined previously [i.e., Clavius and the Coimbra Jesuits]... and it is shown further to be premised on a constructivist conception of mathematical objects themselves. Geometrical figures, according to a dominant line of argument in the seventeenth century, were things *to be drawn* rather than pre-existing in a Platonic realm. [Dear, 1995, pp. 8–9]

While Dear's analysis sheds important light on the background to the development of a major trend in mathematical natural philosophy during the scientific revolution, his study necessarily omits a number of other contemporary approaches to the same problems. One such approach, which is isometric to Dear's study, is the philosophical tradition of nominalism, which was revived in the 17th-century by, among others, Gassendi, and which contributed to epistemological debates concerning the nature of mathematics and mathematical representations of experience. In the case of Gassendi and Hobbes, nominalism led directly to versions of an extreme form of epistemological constructivism, and underlies their respective philosophies of mathematics, which are anti-realist and constructivist.

The term "constructivism" has, however, been applied by scholars to describe a diverse array of practices, mathematical and otherwise. For this reason, I will distinguish between three distinct kinds of constructivism found in early modern natural philosophy. When taken at its most basic level, constructivism in mathematics can refer simply to the technique of generating curves by motion, which was a method used by mathematicians from Archimedes through Newton. This technique—which I will call "mechanical constructivism"—does not have any necessary ontological consequences. A second, more general definition of mathematical constructivism holds that to assert "that there exists a mathematical object (such as a number) with a given property is an assertion that one knows how to find, or construct, such an object."[4] With broader ontological implications than mechanical constructivism, this "mathematical constructivism" opposes a realist philosophy of mathematics by proposing that the properties of mathematical objects are created by the mathematician, and do not have mind-independent existence. Finally, a further definition of constructivism—which I will refer to as "epistemological constructivism"—holds that all products of human understanding (including mathematical objects, words, and species) are artificial constructs of the human mind.[5] This last definition was not widely held during the scientific revolution, but it does apply to the

[4] This definition is found in *The Oxford Companion to Philosophy* [Honderich, 1995, pp. 159–160].

[5] This distinction creates an important demarcation in mathematical ontology: Descartes, for example, had views consistent with mechanical constructivism, but was not strictly a "mathematical" or "epistemological" constructivist.

views, for example, of Gassendi and Hobbes, and is related in important ways to their nominalist epistemologies.

As a distinct tradition in 17th-century philosophy, the influence of nominalism is difficult to define. Very few natural philosophers of the period openly acknowledged adhering to that tradition (Gassendi being a notable exception), but intellectual historians have nonetheless identified a lineage of approaches to epistemological and theological problems in 17th-century philosophy that has clear antecedents in medieval nominalist arguments originated by William of Ockham, John Buridan, and Nicholas of Autrecourt.[6] As a theological argument, nominalism relates most closely to the scholastic distinction between God's absolute and ordained powers: the question concerning whether God's omnipotence grants Him the ability to intervene and change the natural order at any time (*potentia Dei absoluta*), or whether His activity is limited by the order He has established at the moment of creation (*potentia Dei ordinata*).[7] The nominalist axis of this late-medieval debate stressed God's absolute omnipotence, and this literature informed 17th-century voluntaristic theology, which emphasized much the same freedom regarding God's powers of constant intervention [Osler, 1994, Ch. 1]. Among those figures of note for this study who have been associated with 17th-century voluntarist theology are Gassendi, Barrow, and Newton, and elements of their respective voluntarist theological beliefs have been connected in important ways with their natural philosophies [Osler, 1994, pp. 229–236; Osler, 1992; Malet, 1997].

The more familiar aspect of the nominalist tradition, however, is its epistemological and ontological stance toward the existence of Aristotelian "real universals" and Platonic forms. Generally speaking, both the medieval nominalists and their 17th-century counterparts (such as Gassendi and Hobbes) agreed that humans have access only to the singular properties of objects; as a result, the concepts they form are not of real universals, and the representations formed from them are "names" (*nomena*) only. This position is therefore tied to skeptical and probabilistic epistemologies and related more generally to the rise of the broader skeptical tradition in early modern natural philosophy [Popkin, 1960, Ch. 7]. What I will argue distinguishes nominalistic approaches to mathematics from the broader 17th-century skeptical tradition (which includes Descartes, Mersenne, Malebranche, and others) is the belief that, because mathematical symbols, figures, and magnitudes are generalized entities, they are artificial mental constructs which are not ontologically real. Nominalism, therefore, underwrites what I have defined as "mathematical constructivism" in the philosophies of Gassendi, Hobbes, Barrow, and Newton. Furthermore—particularly in the cases of Hobbes and Gassendi—nominalism is the concomitant of the more radical and general position of epistemological constructivism with regard to all objects of knowledge: the belief that knowledge is "constructed" by the mind out of discrete experiences, and therefore does not reflect the necessary ontological reality of the objects it describes. Finally, I will argue that the theological voluntarism associated with both medieval and 17th-century nominalism strongly informed the mathematical philosophies of Barrow and Newton by emphasizing the contingency of mathematical demonstrations on the continued sustenance of the present order of the universe by God. Ultimately, I hope to show that an appreciation for the deep impact of nominalistic ontology

[6] Gassendi openly endorsed nominalism as early as 1624: "You say, do you agree with that mad opinion of the nominalists...? Right, I do; but I believe that I agree with an opinion that is utterly sane" [Gassendi, 1972, p. 43; 1658/1964, Vol. 3, p. 159a]. On medieval nominalism, see Adams [1982, pp. 413–415], Carre [1946, pp. 120–122], Spade [1999], Courtenay [1983, pp. 159–164], and Oberman [1963].

[7] Osler [1994, p. 18] discusses this distinction in detail.

and epistemology helps to clarify the intellectual context of an important tradition in 17th-century mathematical natural philosophy.

Nominalism in Gassendi's philosophy of mathematics

Pierre Gassendi has been widely credited with two major contributions to 17th-century natural philosophy: The first is the revival of the tradition of classical atomism associated with Democritus, Epicurus, and the Roman poet Lucretius, which had wide influence and appeal among the mechanical philosophers.[8] It is clear, for instance, that Robert Boyle was familiar with his works and influenced by his corpuscular philosophy, and there is also evidence that Gassendi's corpuscular system had a significant influence on Newton's early philosophy.[9] The second major contribution was his programmatic promotion of nominalist epistemology and ontology, which reinforced his empirical/mechanical philosophical system. Gassendi has also been noted as a source of the theological voluntarism that was particularly popular in 17th-century England, and his voluntarist conceptions have a firm basis in his ontology [Gassendi, 1972, p. 399; 1658/1964, Vol. 1, p. 280a].[10]

While Gassendi was considered by his contemporaries to be among the leading scientific practitioners—many considered him Descartes' most legitimate rival—he is mostly remembered by historians for his reintroduction of Epicurean atomism. He has certainly not been presented as a philosopher of mathematics of any standing.[11] My purpose here is not to rehabilitate Gassendi's status in this regard, but to highlight his treatment of philosophical issues related to mathematics that are consistent with approaches taken by later figures who most decidedly *were* mathematicians, such as Barrow and Newton. Gassendi's own nominalism, for example, is inextricably fused with his influential version of the mechanical philosophy.[12] His general program of epistemological reform, articulated throughout his lifetime, often centered on mathematical objects and categories. As many of his positions were presented in his public and widely-read commentary on Descartes' *Meditations*, it is reasonable to assert that his views regarding mathematics, as well as his atomism, were known to the general philosophical community [Osler, 1995; Lennon, 1993].

[8] On the revival of atomism in the 17th century, and in particular Gassendi's role in its promotion, see Bloch [1971], Joy [1987], Kargon [1966], and Osler [1985, 1994].

[9] For a discussion of the connections between Gassendi and Boyle, see Osler [1992]. On the subject of Newton's exposure to Gassendist physics, see Westfall [1962, pp. 171–182], and McGuire and Tamny [1983].

[10] According to Gassendi, "atoms are mobile and active from the power of moving and acting which God instilled in them at their very creation, and which functions with his assent" [1972, p. 399].

[11] Several studies of Gassendi have discussed his mathematical interests, but none have seriously considered him a "philosopher of mathematics." Koyré [1957] describes his contribution to mathematics quite dismissively: "To speak of Gassendi's relation to the science of his time might, at first sight, appear to attempt the impossible. And an injustice. Indeed Gassendi was not a great scientist, and in the history of science, in the strict sense of the term, the place to which he returns is not of great importance.... He is not a mathematician." More recent scholarship has, however, taken his mathematical philosophy somewhat more seriously. See Joy [1987] and Brundell [1987]. Osler [1995] does, however, convincingly link Gassendi's nominalism to his rejection of Descartes' realist philosophy of mathematics, but here she focuses more on Gassendi's voluntarist theology than on his nominalist ontology.

[12] Ockham, as well as his later follower Nicholas of Autrecourt, laid the groundwork for Gassendi's nominalist critique of epistemology, and Autrecourt even extended his discussion to a defense of atomism. Osler notes that Gassendi cites the prominent nominalists Gregory of Rimini and Pierre d'Ailly in his *Exercitationes* [Osler, 1994, p. 113].

Gassendi's natural philosophy developed out of an animus towards Aristotelian logic and methodology that spanned his entire career, from his early days as a teacher of logic at Aix-en-Provence to his last appointment, as professor of mathematics at the University of Paris.[13] In his first publication, the 1624 *Exercitationes paradoxicae adversus Aristoteleos*, we get a sense of some of the themes that Gassendi would develop over the next 30 years: Pyrrhonian skepticism and a critique of certainty in language and logic.[14] His 1649 treatise *Animadversiones in decimum librum Diogenis Laertii* was his first major exposition of Epicurus' atomist physics, but in the intervening years, Gassendi had developed an overall system that featured nominalism as a central component linking his epistemology to his ontology [Gassendi, 1649].[15] Among the classical authors Gassendi cites as important early influences are not only Epicurus, but also Pyrrho of Ellis and his much later follower Sextus Empiricus, the former of whose motto "nothing is known" Gassendi adopted in the *Exercitationes*.[16]

Gassendi also maintained an interest in some of the most pressing technical and scientific questions of his day throughout his lifetime. He corresponded extensively with colleagues concerning problems in astronomy and physics, and his *Opera* contains astronomical texts and biographies of several prominent astronomers written between 1631 and 1655. Gassendi was an accomplished astronomer himself, and he corresponded with Kepler regarding eclipses and the motions of Mars and Venus.[17] As early as 1621—several years before the publication of his first work (the attack on Aristotelian logic)—he described his own careful observation of an eclipse in a letter to a patron, Henri du Faur de Pibrac, whom he enlisted for assistance in getting a missive to Kepler.[18] And in 1642 Gassendi published *De motu impresso a motore translato*, in which he was the first to report actual, observational verification of Galileo's famous thought experiment in which balls are dropped from a moving ship, demonstrating the principle of inertia [Gassendi, 1972, p. 120; 1658/1964, Vol. 4, p. 478].[19] Clearly, his interest in scientific epistemology was rooted in practical experience with the sciences, which he attempted to unite in his magnum opus, the *Syntagma Philosophicum* of 1658.

[13] For detailed biographical information about Gassendi's early career, see Jones [1981, pp. 11ff].

[14] "[W]hen I was subsequently burdened with teaching philosophy, particularly Aristotle's, for six full years at the Academy of Aix, I always made it a point that my auditors should be able to defend Aristotle well; but as a kind of appendix to the course I also expounded those opinions which would totally undercut Aristotelian dogmas" [Gassendi, 1972, p. 19; 1658/1964, Vol. 3, p. 100].

[15] Much of the secondary literature about Gassendi concerns his "baptism" of Epicurean atomism to defend against the charge of hedonism and atheism commonly leveled against Epicurus. See Osler [1985, 1994].

[16] "And so I will conclude here with two points as the capstones which will add a certain confirmation of the statement *quod nihil sciatur*" [Gassendi, 1972, p. 97; 1658/1964, Vol. 3, p. 202]. Osler [1994, p. 113] also notes that Gassendi cites the prominent nominalists Gregory of Rimini and Pierre d'Ailly in his *Exercitationes*.

[17] Gassendi published his observations of the transits of Mercury and Venus in *Mercurious in sole visus & Venus invisia, Parisiis, Anno 1631, pro vito & admonitione Kepleri*, reproduced in Gassendi [1658/1964, Vol. 4, pp. 499–511]. Indeed, much of this volume of the *Opera* is devoted to technical astronomy.

[18] Gassendi, Letter to du Faur de Pibrac, April 10, 1621, in Gassendi [1972, pp. 5–8]. He states that he "would note down more carefully the particular things I observed if somebody in your [de Pibrac's] area would give these studies professional attention," indicating that he considers his own attentions "professional" (as opposed to de Pibrac's amateur curiosity).

[19] "On the Way, I enumerated both my own observations and those which Galileo compiled in support of the theorem that 'If the body we are on is in motion, everything we do and all the things we move will actually take place, and appear to take place, as if it were at rest.'... Afterwards, so they would be convinced beyond a doubt, they were taken down to the sea where they were to observe a ship moving with great speed, as well as one at rest, to see whether a stone thrown into the air along the length of the mast... would always keep the same distance from the mast."

Gassendi developed his nominalist position primarily in two major treatises: the 1624 *Exercitationes* and the logical section of the *Syntagma*. Gassendi's first target is the notion that artificial methods of definition that produce categories such as "genus" and "species" somehow illuminate the inner natures of things. In Christianized Aristotelian (or Platonic) philosophy, true propositions may be made because God has composed nature as a hierarchy of essential categories, and logical operations draw on a preordained one-to-one correspondence between the logical structure of the mind and the structure of the world. The "book of nature" is directly translatable, from experience into concepts and from concepts into words. This correspondence exists, quite simply, because God made it so. The scientific investigator assumes and relies on the correspondence *a priori*; he does not create the correspondence through empirical investigation or necessarily verify it through experience.

Gassendi rejects the notion of universal categories entirely. Definitions, according to Gassendi, produce only artificial classes, since they rely on judgements to sort experience into categories. In the *Exercitationes*, he argues "all universality lies in the domain of concepts or words," because "the understanding forms a statement and a predication concerning things as it conceives and names them" [Gassendi, 1972, p. 45; 1658/1964, Vol. 3, p. 160b]. This is a statement echoed in his declaration that there is "no universality outside of thoughts and names" [Gassendi, 1972, p. 43; 1658/1964, Vol. 3, p. 159a].[20] Gassendi's nominalism is also related to his belief that individual experiences of appearances do not produce universal or consistent mental representations of natural objects. This is exemplified in the variance between accounts of a particular phenomenon by different observers:

> since there are so many different appearances of one and the same thing, and since so many different judgements are passed upon it, both by different animals and by different men, as well as by one single man, what other conclusion remains except that we cannot know what anything is like according to itself or its own nature, but only how it appears to some men or to others? [Gassendi, 1972, p. 96; 1658/1964, Vol. 3, p. 202a]

How, Gassendi wonders, can two people be certain that they are describing the same thing when we use particular words to define an object of knowledge? We derive experiences through our senses, but we have no way of knowing what it is like to have another's sight or taste, nor (given disagreements about particular qualities of things) do we have any way of verifying that our own senses are not in error. More importantly, since logic is used to communicate knowledge—and, as Gassendi notes approvingly, according to Cicero logic "make[s] clear what is obscure by translating it into other terms"—Gassendi despairs of finding a common ground on which to base representations of experience.[21] Individual human minds form "mental pictures" of their experiences, but without putting those pictures into words—and exposing them to a further level of translation and distortion—there is no way to communicate them to others. Having thus attacked the foundations of positive knowledge, Gassendi sounds his most pessimistic note: "And so I will conclude here... [with] a certain confirmation of the statement *quod nihil scitur* [that nothing is known]" [Gassendi, 1972, p. 97; 1658/1964, Vol. 3, p. 202b].[22]

[20] "You say, do you agree with that mad opinion of the nominalists...? Right, I do; but I believe that I agree with an opinion that is utterly sane."

[21] Gassendi quotes Cicero on "artificial logic" only to show that it is "useless" [Gassendi, 1972, p. 33; 1658/1964, Vol. 3, p. 150a].

[22] This is the motto of the Pyrrhonists, and also of the 16th-century Spanish natural philosopher Francisco Sanches, whose work *Quod nihil sciatur* was familiar to Gassendi. See Sanches [1988].

An important concept Gassendi develops in this earlier work, which will later relate directly to his discussion of mathematical entities, is his definition of an "idea." An idea, according to Gassendi, is "that image which is present to the mind, indeed is thrust before it almost," which is also called "form," "concept, preconception, anticipation, innate concept... conception and phantasm" by others [Gassendi, 1981, p. 84; 1658/1964, Vol. 1, p. 92a]. It should be made clear, however, that for Gassendi an "idea" is nothing like one of Plato's forms; it does not exist apart from experience, since "every idea which is held in the mind takes its origin from the senses" and "every idea either comes through the senses, or is formed from those which come through the senses" [Gassendi, 1981, pp. 84–85; 1658/1964, Vol. 1, p. 92a].[23] Particular ideas may be grouped together to form compound conceptions in the mind, such as "when from the ideas of mountain and gold [i.e., the *impressions* of those physical objects] the mind pictures the idea of a mountain of gold; when from the ideas of man and horse it gets the idea of centaur... and so on," a process Gassendi terms "unification." Ideas can also be connected to form general categories, as when the mind takes individual ideas and "by examining the ideas individually and separating out that which all have in common, at the same time disregarding or ignoring mutual differences, takes as a common, universal and general idea that which has been thus abstracted and which contains nothing that is not common... and this is what is termed the genus" [Gassendi, 1981, pp. 85–87; 1658/1964, Vol. 1, pp. 93a–b].

Consistent with his nominalistic principles, however, Gassendi asserts that these universal or general ideas are *not* ontologically real entities: "all the things which are in the world and which are able to strike the senses are singulars" [Gassendi, 1981, p. 85; 1658/1964, Vol. 1, p. 92b]. Although the mind constructs compound and general ideas of objects, this is an artificial process. This does not, however, lead Gassendi to reject the possibility of obtaining knowledge. Gassendi quotes the Stoic dictum *quod nihil scitur* only to reinforce his claim that "men do not know the inner nature of things, or their so-called real essences"; it is basically a rhetorical device, designed to point out the limits of human knowledge, not its impossibility [Gassendi, 1972, p. 97; 1658/1964, Vol. 3, p. 202b]. Gassendi's solution to this problem is essentially to propose an extreme form of epistemological constructivism: abstracted general categories have some value in philosophy because they are constructed by the human mind, and therefore may function in logical propositions whose rules are defined artificially.

Gassendi extends this notion to mathematics as well. Later in the *Exercitationes*, he concludes "that whatever certainty and evidence there is in mathematics is related to appearances, and in no way related to genuine causes or the inner natures of things" [Gassendi, 1972, p. 107; 1658/1964, Vol. 3, p. 209a]. This is the same caution he makes when he discusses the limits of knowledge in language and logic: propositions may only be said to be certain or true insofar as they reflect our imperfect perceptions of empirical data, and not in relation to some absolute standard of truth or existence.[24] Indeed, as Gassendi continues, mathematics must be explicitly concerned with matters of experience, since "the moment you pass beyond things that are apparent, or fall under the province of the senses and experience, in order to inquire about deeper matters, both mathematics and all other branches of knowledge become completely

[23] This is a familiar quotation from Aristotle, who also suggested an empirical basis for ideas. Gassendi departs from Aristotle, however, in denying that even ideas correspond "naturally" with real universal categories.

[24] Gassendi was certainly aware of the Renaissance debate over the certainty of mathematics involving Clavius, Piccolomini, Barozzi, Pereyra, and others. He quotes, in the *Exercitationes*, from Pereyra's 1576 *De communibus omnium rerum naturalium principiis et addectionibus libri quindecim*, and as Manucosu notes, "passages from Gassendi and Barrow on the nature of mathematics are incomprehensible without referring back to the *Quaestio*" [1996, p. 92].

shrouded in darkness" [Gassendi, 1972, p. 107; 1658/1964, Vol. 3, p. 209a]. Clearly Gassendi does not extend special epistemological privileges to mathematics, nor does he grant mathematics a unique role in scientific investigation; mathematics, like all other branches of knowledge, must be concerned only with knowledge of appearances and individuals. This means that Gassendi's view of mathematics is, like his view of language, nominalistic. Mathematics, being a human interpretive faculty, seeks to make general statements about objects of knowledge. Because, however, the mind only has access to individuals, that generalizing process is artificial. Just as words are general terms—based on the experience of individuals—that have no real existence in nature, so must mathematical objects be "considered in actual things," since "as soon as numbers and figures are considered abstractly (a way they have never been in existence), then they are nothing at all" [Gassendi, 1972, p. 107; 1658/1964, Vol. 3, p. 209a].

Gassendi concludes his discussion of mathematics in the *Exercitationes* by noting that a geometrical object, such as a triangle, is considered by the mind to be a general class even though it is based on knowledge of particular triangles. As Gassendi states,

> if mathematics makes some proof, for instance about the triangle, it does not name this triangle or that one; but yet it understands this triangle and that one, not singly, but in conjunction with all others. In fact, however, if it did not base its conclusions upon triangles appearing in some material form, it would only be chasing chimeras since no other triangles but these can exist. [Gassendi, 1972, p. 108; 1658/1964, Vol. 3, p. 209a]

For geometry to have any usefulness it must, according to Gassendi, deal with general classes of objects, but paradoxically geometrical objects only derive their validity as they are observed in actual objects by the senses. It is the same with Gassendi's conception of language: words exist only as they are abstracted from sensory data, but that very abstraction prevents them from ever capturing things as they "actually are." Gassendi (and, in addition, Hobbes) argued that because mathematics relied on abstraction it was essentially reduced to a "grammar" for representing imperfect perceptions of physical objects. This position reflects Gassendi's commitment to a form of mathematical constructivism, which attributed a great deal of "certainty" to mathematics, but whose certainty was granted precisely because mathematical rules and objects were believed to be artificial. Gassendi's nominalism and mathematical constructivism thus departed from Cartesian and Galilean philosophy of mathematics by denying that mathematical objects (and similar abstractions) were ontologically real.[25]

Gassendi's interest in the epistemological and ontological foundations of mathematics continued in the years following the *Exercitationes*. Lynn Joy describes Gassendi's involvement in a debate over the existence of indivisible magnitudes that circulated among a group of philosophers close to the Minim friar Marin Mersenne in the mid-1630s. Whereas most of the participants chose to analyze indivisibles as if they were mathematical points, Gassendi argued from the position of physical atomism, and his concerns about mathematical abstraction are in evidence. Joy quotes from a letter to Mersenne in which Gassendi raises several doubts about the validity of applying mathematical demonstrations to solve what he considers a purely physical problem, and she states that Gassendi's skepticism about the reality of abstractions separated him from the other participants in the debate [Joy, 1987, pp. 91–92].[26] Joy also connects this attitude to Gassendi's nominalism, and cites Gassendi's belief that "anyone attempting to give a mathematical description of a physical state of affairs should always bear in mind the fact that

[25] Galileo's mathematics has, now somewhat infamously, been classified as "Platonic" by Koyré [1978], a characterization that has been questioned by more recent scholarship, including Machamer [1998] and Feldhay [1998]. Their reinterpretations do not, however, deny that Galileo believed mathematical entities were fundamentally real.

such a description is applied to individual physical objects with varying degrees of accuracy" [Joy, 1987, p. 102].[27]

Gassendi's response to Mersenne in this instance offers not just the standard empiricist objection that the senses are superior to the intellect, but also the more penetrating skepticism about the existence of mathematical objects at all. As Gassendi explained his position, "I mention this [the debate over Ptolemaic versus Copernican systems] only in order that you understand that I say nothing absurd when I hold as mere hypotheses the points, lines, and surfaces defined by the mathematicians—every one of which could be made about nonexistent things."[28] Gassendi is claiming here that the use of general classes of objects in geometrical demonstrations is justified only by the definitions given to those objects, definitions that are constructed in the human mind, and which are therefore suspect in solving physical problems. Just as according to Gassendi a word that signifies something outside of one's experience has no physical referent, a geometrical object (such as a point) that has no basis in experience cannot be used to signify an actual physical state of affairs.

Gassendi's mathematical philosophy is also evident in his published objections to Descartes' *Meditations on First Philosophy* in 1642. Among the group of scholars invited by Marin Mersenne to contribute to this project (which included Mersenne himself, Hobbes, and the logician and grammarian Antoine Arnauld) Gassendi provided the longest and most critical response, which provoked the harshest rejoinder from Descartes. Descartes, like Gassendi, drew close parallels between the mental operations performed in logical and linguistic arguments and those in mathematics. The difference, of course, is that Descartes believed in the reality of mental abstractions, and in the innate correlation between those representations and the physical world. In his examples, Descartes turns explicitly to mathematics and geometry for evidence of the certainty of human cognition; indeed, mathematics is the paradigm for such examples, as it displays the power of the human mind. Descartes claims, for example, that regardless of the physical existence of any perfect geometrical figure (such as a triangle), because he can imagine that figure having certain determinate properties, there must be "a determinate nature, or essence, or form of the triangle which is immutable and eternal, and not invented or dependent on my mind" [Descartes, 1984, Vol. 2, p. 44]. This suggests that not only are humans able to clearly and distinctly perceive the essences of things (even incorporeal things), they are also able to conceive and represent those essences without distorting them. Descartes responds dismissively to the counterarguments of his critics:

> many people of great intelligence [i.e., Gassendi] think they clearly see that mathematical extension, which I lay down as a fundamental principle of my physics, is nothing other than my thought, and hence that it does not and cannot have any subsistence outside of my mind, being merely an abstraction which I form from physical bodies. And they conclude that the whole of my physics "must be imaginary and fictitious, as indeed the whole of pure mathematics is, whereas real physics dealing with things created by God requires the kind of matter that is real, solid and not imaginary." [Descartes, 1984, Vol. 2, pp. 274–275]

[26] As Joy notes, Gassendi incorporated a "strong bias against introducing mathematical arguments into his answers and his doubts about the usefulness of determining the properties of strictly mathematical points were not shared by the majority of Poysson's nine other correspondents."

[27] Joy also states that Gassendi believed a mathematician "identifies physical points and magnitudes and attributes the appropriate names to these individuals whenever he can show that their physical properties adequately resemble the properties specified in the definitions of their names."

[28] Gassendi to Mersenne, December 13, 1635, translated in Joy [1987, p. 103].

Gassendi's objection to this line of reasoning is based solidly in his nominalist principles: he claims that while a figure having three angles and three sides "will not fail to be a triangle" simply because that is the word's definition, the word "triangle" signifies nothing without being tied to physical experience: "things not yet created and having no existence, but being merely possible, have no reality and no truth" [Gassendi, 1972, p. 253; 1658/1964, Vol. 3, p. 377b]. Gassendi also doubts Descartes' ability to have "understanding" of more complicated geometrical figures (such as the chiliagon, a thousand-sided figure) in the way he has claimed, and he accuses Descartes of confusing "imagining" with "understanding." Gassendi grants that one may call up a mental picture of a triangle, and "understand" that this figure is connected with a particular word and definition, but he denies that Descartes can "grasp distinctly" in his mind a thousand-sided figure, or that he can truly understand whatever tortured representation of that figure his mind is able to create. At most, Gassendi argues, one can "perceive that the word "chiliagon" signifies a figure with a thousand angles"; however, "that is just the meaning of the term, and it does not follow that you *understand* the thousand angles of the figure any better than you *imagine* them" (Gassendi, in Descartes [1984, Vol. 2, p. 229]). Ultimately, Gassendi concludes, all knowledge—including mathematical knowledge—must be tied to the experience of particulars in order to be considered to have real existence. In a denial of one of Descartes' stated principles of physics, Gassendi announces that "the subject matter of pure mathematics—including the point, the line, the surface, and the indivisible figures which are composed of these elements and yet remain indivisible—cannot exist in reality" (Gassendi in Descartes [1984, Vol. 2, p. 228]).

From his response, it is clear that Descartes recognized the philosophical basis for Gassendi's attack, though he disingenuously pretends that Gassendi's "purpose has rather been to bring to my attention the devices which might be used to get round my arguments by those whose minds are so immersed in the senses that they shrink from all metaphysical thoughts" [Descartes, 1984, Vol. 2, p. 241]. Later, in response to Gassendi's objections to his philosophy of geometry, Descartes argues that

> unless you are maintaining that the whole of geometry is also false, you cannot deny that many truths can be demonstrated of these essences; and since they are always the same, it is right to call them immutable and eternal. The fact that they may not accord with your suppositions about the nature of things, or with the atomic conception of Democritus and Epicurus, is merely an extraneous feature which changes nothing; in spite of this they undoubtedly conform to the true nature of things established by God. [Descartes, 1984, Vol. 2, p. 262]

Gassendi himself recognized that this split was fundamental, and mockingly queried Descartes whether "just because these and other splendid and Platonic things have been said, have they been splendidly and truly proven?" [Gassendi, 1972, p. 257; 1658/1964, Vol. 3, p. 378b]. This divide helps explain the failure of Gassendi and Descartes to conduct a meaningful exchange of views.[29] Descartes' explanation for the efficacy of mathematical demonstrations was rooted in what is essentially metaphysical realism, which diametrically opposed Gassendi's nominalism.[30]

[29] Joy [1987] has argued that the two philosophers "talked past" one another because of methodological differences—Descartes was unable to parse Gassendi's logical critiques and humanist methods. But from my reading of this exchange, I think the problem was more fundamental. In this I am inclined to follow Lennon [1993], who maintains that Gassendi's conception of "the identity of idea and object demands an ontology incompatible with Descartes's mathematical realism, his Platonic account of the material world."

[30] In a letter to Mersenne written in 1630, Descartes had stated his views about the "reality" of mathematics quite bluntly: "But in my physics I will not touch on certain metaphysical questions, particularly this one: That the truths of mathematics,

The legacy of nominalism and mathematics

Hobbes, nominalism, and constructivism

If I were attempting to argue that Gassendi inaugurated a particular "school" of mathematical philosophy, it would be fair to ask whether Gassendi's nominalistic mathematics itself had any real or lasting impact on later beliefs and practices. Although his philosophy was certainly not without influence—Gassendi was read and admired by Hobbes, Barrow, and Newton, whose positions will be examined presently—I am not arguing for a direct chain of causal influence between Gassendi and these other natural philosophers. Rather, I argue that Gassendi's position on foundational questions in the philosophy of mathematics is representative of a distinctive approach to these questions during the 17th century—an approach which is hallmarked by commitments (in varying proportions, to be sure) to nominalism, mathematical/epistemological constructivism, and theological voluntarism. This constellation of beliefs had a currency in 17th-century intellectual culture that was broader than the views of any one particular philosopher, but it has not been linked explicitly to mathematics in scholarly literature.

In the case of Hobbes, more than one historian has remarked that his theory of cognition and representation is explicitly nominalistic. According to the historian of linguistics G.W. Padley, Hobbes was "the exemplar of this tendency to extreme Nominalism" which has its roots in the skeptical philosophy of the late scholastic period, "which Hobbes did more than any other philosophical writer to hand on to modern times" [Padley, 1976, pp. 141–142]. Padley's assessment is supported by Douglas Jesseph, who contends that Hobbes's philosophy "goes beyond the nominalism of his predecessors," including that of the "original" nominalist, William of Ockham [Jesseph, 1999, pp. 208–209]. Indeed, Hobbes's theory of language and its role in concept formation is quite similar to Gassendi's, and his nominalism is explicit. As he writes in *Leviathan*, although some names "are common to many things," there is "nothing in the world Universall but Names; for the things named, are every one of them Individuall and Singular" Hobbes [1996/1651, p. 26].

Hobbes's nominalism leads him to ascribe an essentially conventional and linguistic definition to the concept of "existence," and he claims in his own objections to Descartes' *Meditations* that

> if it turns out that reasoning is simply the joining together and linking of names or labels by means of the verb "is". . . . It would follow that the inferences in our reasoning tell us nothing about the nature of things, but merely about the labels applied to them; that is, all we can infer is whether or not we are combining the names of things in accordance with the arbitrary conventions which we have laid down in respect of their meaning. (Hobbes, in Descartes [1984, Vol. 2, p. 125])

Language, Hobbes claims repeatedly, provides no knowledge of the essences of things, and his empiricist theory of cognition holds that concepts are formed out of the "decaying" images of individual objects

which you call eternal, are established by God and are entirely dependent on Him as are all the rest of the creatures [tout le reste des creatures]. It is in fact the same to speak of God as a Jupiter or Saturn who are subject to [the river] Styx and to destiny, as to say that these truths are independent of Him. Do not fear, I urge you, to argue and publish everywhere, that it is God who establishes these [mathematical] laws in nature, just as a king establishes the laws in his kingdom. Now there is no one [law] in particular that we cannot comprehend if our intellect makes itself consider it, and they are born in our minds [mentibus nostris ingenitae] as would a king imprint his laws in the hearts of all his subjects, if he had the power to do it." (Descartes to Mersenne, 15 April 1630, in Mersenne [1945, Vol. 2, p. 431].)

provided by the senses, as he argues in *Leviathan* and elsewhere [Hobbes, 1996/1651, pp. 15–16]. In identifying algebra with natural languages, Hobbes focuses his well-documented antagonism toward symbolic analysis on the claim that the symbols of the algebraists (like words) are purely conventional.[31] Hobbes view of both language (and cognition) and mathematics was also broadly constructivist: as Pycior notes, "in his *Elements* Hobbes turned to universal names rather than general abstract ideas to explain the human ability to generalize" [Pycior, 1997, p. 140]. Similarly, in his philosophy of geometry (which he regarded as a more genuinely demonstrative science than algebra and arithmetic), Hobbes argues that the basis for human knowledge is grounded in the fact that geometrical objects are constructed by humans. "Geometry therefore is demonstrable," Hobbes writes, "for the lines and figures from which we reason are drawn and described by ourselves" [Hobbes, 1839–1845, Vol. 7, p. 184]. Geometry, he asserts, does indeed present generalized concepts, but they are general only insofar as they are drawn by human beings, and not because they reflect real universals.

This leads Hobbes to develop a what can be termed a "mechanical geometry" that is founded on multiple levels of constructivism. Motion is the basis for Hobbes's materialist geometry and is the foundation for analytical methods in mathematics, since Hobbes originally conceived his new geometry as "analysis by motions or the method of motion" [Hobbes, 1839–1845, Vol. 7, p. 307]. To this extent, Hobbes's philosophy of geometry may be described as "mechanical constructivism":

> The paths of motions simpliciter (in which geometry consists) ought to be investigated in the first place, and then the paths of motions generated and obvious, and finally the paths of motion internal and invisible (which physicists study). Thus those who study natural philosophy study in vain unless they take their principles of inquiry from geometry; and those who write or lecture about such things and are ignorant of geometry abuse their readers and listeners. [Hobbes, 1981/1655, p. 301]

Whereas Descartes, for example, believed geometry to be the foundation of physics because it offered insight into nature beyond what was perceivable by the senses and expressible in mere language, Hobbes's geometry was rather an extension of his general, nominalistic and empiricist philosophy of representation, which involved mathematical and epistemological constructivism as well. For Descartes, geometry was generalizable because geometrical relations were installed by the creator, and because its universal truths were directly perceived by the mind. Hobbes, on the other hand, allowed geometrical definitions and statements to have general force by the same principle that made statements in language general: as human constructions they were based in experiences of individual phenomena, and were not eternal truths abstracted from the essences of things [Pycior, 1997, p. 140].

Hobbes conceived the objects of geometry to be representations of the motions of certain basic objects. As he notes in *De Corpore*, "*Lines*, *superfices*, and *solids*, are exposed, first, by *motion*... but so as the marks of such motion be permanent; as when they are designed upon some matter, as a line upon paper; or graven in some durable matter" [Hobbes, 1839–1845, Vol. 1, p. 140]. A line, in other words, is the path traced by a point in motion; a plane (or "superfice") is the surface swept out by a moving line, etc. A geometrical representation, therefore, is not the abstracted essence of an object or a Platonic form, but rather a kind of "time-exposure" tracing the path of a geometrical object or objects in motion over individual moments. This is not necessarily to say that Hobbes imagined geometrical shapes to be literally "in motion"; rather, he treated geometrical objects as if they represented motions, because he recognized geometry as valid only insofar as it described actual physical phenomena. Geometry, then,

[31] On Hobbes's mathematics, see Jesseph [1999] and Grant [1996].

was a language for expressing physical relationships. Since Hobbes's ontology was mechanical—and since he denied the existence of "abstract essences" which could be separated from matter—geometry must be a "mechanical" science in order to properly represent nature.

Hobbes's nominalism is also demonstrated by his well-known animus toward the symbolic notation of analytic geometry and algebra, over which he famously (and somewhat rashly) engaged the hostile attention of the mathematician John Wallis.[32] His objection to these methods essentially relates to his concern about the ontological content of the entities the symbols purport to represent. In Hobbes's scheme there is a very real concern with the decay of information through repeated "translations" from experience to concepts, and his positive approach to science involves devising methods to minimize this loss of information by fixing precise definitions and signification onto words used. Though he believed the true essence of a thing would never be captured, Hobbes hoped nonetheless that through precise conventional definitions humans might have a greater understanding of what they are talking about when they describe nature. In the case of analytic geometry, however, there is an extra translation made that Hobbes sees as unnecessary and damaging:

> For the conception of the lines and figures (without which a man learneth nothing) must proceed from words either spoken or thought upon. So that there is a double labour of the mind, one to reduce your symbols to words, which are also symbols, another to attend to the ideas which they signify. [Hobbes, 1839–1845, Vol. 7, p. 330]

Since geometrical figures are already translations of experience into symbols, he finds no reason to take the step of an additional translation into further symbols, which only makes their original signification more uncertain. It is the principle of parsimony that drives this critique, but it is drawn from his general theory of meaning.

Hobbes was not, however, claiming that symbolic notation was entirely useless. He considered all reasoning to be in a sense symbolic, since marks are symbols (or signs) of concepts, and believed there was no possibility of reasoning on actual "things" (as there was for Descartes). Rather, the issue was again to find the most accurate and secure symbolic techniques, and to this end he did allow that symbolic shorthand was acceptable for the experienced geometrician to briefly note his discoveries to himself. In *Six Lessons to the Professors of Mathematics*, though, Hobbes made it clear that he thought symbolic geometry was not fit for public demonstrations: "Symbols are poor unhandsome, though necessary, scaffolds of demonstration; and ought no more to appear in public, than the most deformed necessary business which you do in your chambers" [Hobbes, 1839–1845, Vol. 7, p. 248]. Hobbes's perhaps excessive rhetoric must be understood here in the context of his extremely acrimonious debate with Wallis, to whom he was addressing that passage, but the point nonetheless is that symbols place an unnecessary burden on communication. Since the most essential acts of communication in geometry are presenting persuasive demonstrations and instructing students, Hobbes felt that analytic geometry was far inferior to traditional methods because traditional methods were easily comprehended by all. Hobbes goes so far as to suggest that nothing new can be discovered through analytic geometry, stating that Wallis "mistook the study of *symbols* for the study of *geometry*, and thought symbolical writing to be a

[32] See Jesseph [1999] for a detailed analysis of this debate. Unfortunately, there is a lack of secondary literature on the connection between algebra and theories of language in 17th-century philosophy, making it difficult to situate nominalistic critiques of mathematics like Hobbes's within a broader linguistic tradition. This is, hopefully, a topic that will be addressed in future scholarship.

new kind of method, and other men's demonstrations set down in symbols new demonstrations" [Hobbes, 1839–1845, Vol. 7, pp. 187–188].

The question remaining, then, is why Hobbes thought "traditional" geometry was more secure than algebra and analytic geometry. Despite questions about deficiencies in his technical abilities, the epistemological principles in Hobbes's mathematical philosophy are consistent with his overall treatment of language and signification, so I suggest he should be taken seriously as a philosopher of mathematics, if not always as a mathematician. We have already seen that Hobbes's philosophy of mind privileges not the mind's inherent capacity to penetrate nature, but rather the faculty of the understanding to construct knowledge based on the marks and signs drawn from experience. Certainty comes only when the objects being discussed are conventionally defined, since then their precise meanings may be known. "Certainty" is thus distinguished from "ontological reality."

This scheme also applies to geometry, as Hobbes explains in the dedication to *Six Lessons*:

> the science of every subject is derived from a precognition of the causes, generation, and construction of the same; and consequently where the causes are known, there is place for demonstration, but not where the causes are to seek for. Geometry therefore is demonstrable, for the lines and figures from which we reason are drawn and described by ourselves. [Hobbes, 1839–1845, Vol. 7, p. 184]

Because of his nominalism, Hobbes believes that all general names and definitions are conventional, so truth can only be assigned insofar as statements made about general classes agree with the recognized definitions applied to those objects. For Hobbes, geometry is entirely dependent as a science on the application of conventional definitions: "as they, who handle any one part of geometry, determine by definition the signification of every word... so must he which intendeth to write a whole body of geometry, define and determine the meaning of whatsoever word belongeth to the whole science" [Hobbes, 1839–1845, Vol. 7, p. 192]. Geometry has certainty precisely because it uses axioms and definitions that are defined by humans, but that certainty applies only insofar as its universal objects—like geometrical figures—are "constructed." As Jesseph notes, according to Hobbes such certainty is possible only in pure mathematics, since the physical world does not present causes to our understanding. Thus, geometry is "maker's knowledge," since its certainty is grounded "in our construction of the objects known" [Jesseph, 1999, pp. 220–221].[33] Hobbes's philosophy can, therefore, be seen to be underwritten not only by mechanical and mathematical constructivism, but also by a general epistemological constructivism that—much like Gassendi's—is drawn from his commitment to a nominalist ontology.

Isaac Barrow: nominalistic arithmetic and voluntarist geometry

Despite the animadversions of renegades like Hobbes, by the 1670s, mathematical practitioners had become increasingly committed to the use of analytic geometry, infinitesimal techniques, and algebraic notation, all of which added greatly to the power of mathematical natural philosophy, but which also presented a number of epistemological and ontological questions.[34] These questions concerned the nature and reality of mathematical objects, the truth of mathematical demonstrations, and the proper "language" for mathematical discourse. Mathematical practitioners developed a number of strategies for justifying

[33] Jesseph is referring explicitly to Antonio Perez-Ramos's idea of maker's knowledge developed in Perez-Ramos [1988]. I accept the term, however, with reservation, because I do not necessarily place Hobbes's "constructivist" epistemology in the tradition of artisanal craftsmanship Perez-Ramos assigns to Bacon.

the use of increasingly abstracted and formalized analytic techniques, but as a number of historians have shown, the transition from a mathematics oriented towards traditional Euclidean geometry to one centered on analysis was by no means uncontroversial.[35] The tension between Hobbes and Wallis, for example, was felt (albeit often less dramatically) by many others in the wider mathematical community, and particularly in Britain discussions of the "usefulness" and legitimacy of analysis continued into the 18th century.[36]

The traditional paradigm established by Burtt and Koyre, which saw a progressive rise in a homogeneous tradition of "mathematization" through Galileo, Descartes, and Newton, is no longer sufficient to explain this development. As Michel Blay has argued, it is more accurate to describe two distinct ontological approaches to mathematical philosophy in this period, one based on a realist geometrical ontology, which he terms "geometrization," and the other based on analytical techniques, which he terms "mathematization." According to Blay, the distinguishing feature of mathematization is that mathematical entities themselves are not ontologically real; rather, its "object is to reconstruct the phenomena of nature within the domain of mathematical intelligibility in such a way that these phenomena find themselves governed by quantitative laws which can be exploited for the purpose of predicting the course of nature by means of mathematical reason" [Blay, 1998, p. 3]. Such reconstruction is consistent with the definition of mathematical constructivism I presented earlier, which implies that the mathematical construction of nature is distinct from some real ontological order. In the cases of two important mathematical practitioners of the later seventeenth century—Barrow and Newton—this approach to mathematical constructivism is linked to a "physicalist" conception of geometrical magnitudes, and also, by extension, to important elements of nominalist ontology.

Barrow was a dedicated supporter of the synthetic geometrical approach, and he argued at length against the validity of analytical methods.[37] His arguments were, however, targeted at the ontological basis for analytic geometry, and his objection to the reality of algebraic demonstrations has a significantly nominalistic basis.[38] Barrow's philosophy of geometry incorporated elements of mechanical, mathematical, and epistemological constructivism, and his philosophy of arithmetic was quite explicitly nominalistic. His characterization of mathematics as "constructed" knowledge is also similar to Gassendi's conception of mathematics as an artificial and descriptive language about nature.

In a series of lectures given at Oxford between 1664 and 1666, and later published as *The Usefulness of Mathematical Learning*, Barrow outlined a clear and distinctive philosophy of mathematics that incorporates nominalism, constructivism, and theological voluntarism. Writing on the ontological

34 There are a number of excellent sources on the history of algebra and analysis. A few of the most important are Boyer [1956, 1959], Cajori [1929], Klein [1968], and Kline [1972]. More recent contributions include Bos [1991], Blay [1998], and Pycior [1997], among others.

35 Pycior [1987, 1997], Hill [1996], and Jesseph [1993, 1999], for example, have stressed this point.

36 Perhaps the most prominent early 18th-century example is George Berkeley's critique of Newtonian calculus in his treatise *The Analyst* [1992]. As Jesseph [1993, pp. 94–95 and 118–120] has argued, Berkeley's objection to the method of fluxions has a strongly nominalistic flavor.

37 On Barrow's mathematical methods, see Mahoney [1990].

38 Nominalism was only one of several influences on Barrow's mathematical philosophy. Barrow began his career as a classical scholar, and he produced an edition of Euclid's *Elements* in 1656. One of the chief 17th-century sources for Euclid was Proclus's Platonic commentary on *Elements*, and it is clear that Barrow made use of this resource. It is unclear, however, how deeply he was influenced by the Platonic aspects of this commentary, since major aspects of Barrow's mathematical philosophy are incompatible with Platonism, as argued below. On Barrow's intellectual development, see Feingold [1990, pp. 37–47].

foundations of arithmetic, he explicitly contrasts his own conception with contemporary examples of rationalist philosophy: "There is no need," he writes, "for supposing any *physical Anticipations*, *common Notions*, or *cogenite Ideas*" in the mind or "to invent, or devise any *Platonic Remembrances*, and I do not know what Resuscitations of the Mind as asleep" [Barrow, 1734/1970, pp. 115–116]. This discussion includes a definition of "number" that is redolent of the kind of nominalism expressed by Gassendi and Hobbes. Barrow writes that "a *Mathematical Number* has no Existence proper to itself, and really distinct from the Magnitude it denominates, but is only a kind of *Note* or *Sign* of Magnitude considered after a certain Manner" [Barrow, 1734/1970, p. 41]. Here he associates the symbols of mathematics with linguistic and logical operations, and draws a clear demarcation between what such symbols signify and the true "essences" of things. Like Hobbes, Barrow also accepts that geometry has more demonstrative validity than arithmetic, but he stresses that this is because geometrical demonstrations can be tied to specific physical instances. Taking the example of the simple equation $2 + 2 = 4$, Barrow points to what he views as a central shortcoming in arithmetic:

> Whence comes it to pass that a Line of two Feet added to a Line of two Palms cannot make a Line of four Feet, four Palms, or four of any Denomination, if it be abstractedly, i.e., universally and absolutely true that $2 + 2$ makes 4?... from whence I infer that $2 + 2$ makes 4, not from the abstract Reason of the Numbers, but from the Condition of the Matter to which they are applied. [Barrow, 1734/1970, p. 37]

Geometry, on the other hand, provides statements that are about physical magnitude, which is tangible and known to us through experience, and here Barrow's constructivism surfaces. Barrow claims that "every Sensation is of Things particular, and from the Existence of a particular Thing, we may conclude that something like it may exist, but not that it actually does exist, as has sometimes been taken notice of before," which implies a general mathematical constructivism towards geometrical objects [Barrow, 1734/1970, p. 169]. By endorsing a physicalist philosophy of geometry, he also demonstrates a clear preference for mechanical constructivism. In another series of lectures, the *Lectiones geometricae*, he writes that the objects and figures of geometry are ultimately reducible to rectilinear motions:

> For every line that lies in a plane can be generated by the motion of a straight line parallel to itself, and the motion of a point along it; every surface by the motion of a plane parallel to itself and the motion of a line in it (that is, any line on a curved surface can be generated by rectilinear motions); in the same way solids, which are generated by surfaces, can be made to depend on rectilinear motions. [Barrow, 1916, p. 49]

In his discussion of this subject, however, both in *Lectiones geometricae* and *Usefulness*, Barrow introduces an additional wrinkle which makes comparison to Gassendi and Hobbes more compelling: a combination of general mathematical *and* epistemological constructivism with theological voluntarism. According to Barrow, objects that can be described by mathematical demonstrations but which do not exist in the actual physical world may also be said to be potentially real, since in having no inconsistencies they could possibly be created by God at any time. Antoni Malet has described this feature of Barrow's philosophy, noting that "Barrow's first principles need not be self-evidently true—they only must be free from contradiction." According to Malet, mathematical demonstrations may produce logically coherent descriptions of "worlds" that do not correspond with reality as we perceive it, but rather which "describe 'imaginary' worlds which God might create" [Malet, 1997, pp. 277–278].

Barrow uses this notion to defend the mathematical coherence of the several different cosmological doctrines that had been advanced over the centuries. He notes that the theories of Ptolemy, Copernicus, Tycho, and Kepler are equally valid from a purely mathematical point of view, though they do not all present an accurate picture of the actual physical world. "I say," writes Barrow,

> though I imagine all these to be false, at least unknown in respect of those Stars; yet because nothing hinders, but God may create such a World, where the Stars will exactly agree with such Motions; therefore the Demonstrations depending upon such Hypotheses are most true, and their Astronomy true, not indeed of this World, but of the other, which is supposed capable of being created by God. [Barrow, 1734/1970, p. 111]

This idea of the potential existence of other worlds that are mathematically consistent is tied, according to Malet, to Barrow's theological voluntarism, which posits that God has the freedom to act in the world to change the order of nature at any time. This subject has been much discussed in recent years, most notably in connection with the atomistic philosophy of Gassendi and Charleton, and with the theological basis for Newton's concepts of force and universal gravitation. [Osler, 1994; Malet, 1997; Davis, 1996]. It is clear that Barrow subscribes to at least a form of voluntarism, as he holds that "every Action of an *efficient Cause*, as well as its consequent *Effect*, depends upon the *Free-Will* and Power of *Almighty God*, who can hinder the Influx and Efficacy of any *Cause* at his Pleasure." Barrow grants that God's intervention may disrupt the normal pattern of cause and effect, using the example of "the necessary Existence of Fire inferred from Ashes or Smoak" to question whether one can doubt "but God can immediately create Ashes and Smoak, or produce it by other Means" [Barrow, 1734/1970, pp. 88–89].

Leaving aside the question of whether Barrow believed God actually does intervene in the causal structure of nature, the potential for such action raises at least a shadow of skepticism about the certainty of human understanding. But rather than take this potential uncertainty to be limiting, Barrow seems to have found it liberating: Barrow can treat mathematical representations as if they were "real," without having to take the step of professing ontological realism. Thus, Barrow can argue that "though no such Notions be ever found in the Nature of Things, as Geometricians suppose to be described by *Spiral Lines, Quadrices, Conchoids, Cissoids,* &c.," so long as they are consistent with mathematical rules they may be assumed to be "virtually" real, since they "follow from such Suppositions, [as] are rightly demonstrated" and therefore could exist if God willed it [Barrow, 1734/1970, p. 111]. In a sense, Barrow is privileging the intellect, but he is not proposing an abstract Platonic realm of ontologically real idealized objects. Rather, he is suggesting that mathematicians are not to be limited by the boundaries of what is physically possible in the real world when constructing virtual "alternate realities" that are inhabited by fictional objects. Barrow explicitly maintains the distinction, however, between the real and the merely possible, and does not attach this belief to a doctrine of corresponding levels of reality in an ontological hierarchy. Just as these fictional entities might be created by God in material substance, they are also literally constructed in the mind of the mathematician, since "the Domains of Reason do far exceed the Limits of Nature; the intelligible World is vastly farther extended and more diffusive than the sensible World, and the Understanding contemplates many more Things than the Sense" [Barrow, 1734/1970, pp. 111–112].

Still, as Malet points out, in order for mathematics to contribute to practical natural philosophy, "theories need testing to ensure that they apply to *this* world" [1997, p. 279]. The obvious empirical strategy is to proceed by inductive reasoning and to compare mathematical results with data collected from many observations or experiments. But evidence gathered by induction is limited by the caveat that no matter how many particulars may be observed, there is no way to assure that comparisons are complete or that theories can be verified with absolute assurance. Here Barrow's "many worlds" philosophy provides an interesting exemption from this limitation: rather than assuming that induction must be complete, he requires only that enough observations be made to predict the possibility that a general rule may be posited. Barrow claims that "every Sensation is of Things particular, and from the Existence of a particular Thing, we may conclude that something like it may exist, but not that it actually does exist, as has sometimes been taken notice of before" [Barrow, 1734/1970, p. 169]. In other words, Barrow combines the nominalist attitude that we may only experience particulars with the belief that general knowledge may be acquired about the natural world. But he avoids radical skepticism by suggesting that general features may be extrapolated from individual observations, while also avoiding essentialism by claiming that such generality is only probable. This is reflected in Barrow's claim that "the Truth of Principles does not solely depend on *Induction*, or a perpetual Observation of Particulars," but rather on a modified empirical method in which "only one Experiment will suffice (provided it be sufficiently clear and indubitable) to establish a true Hypothesis, to form a true Definition; and constitute true Principles" [Barrow, 1734/1970, p. 116]. It should be remembered that Barrow's notion of a "hypothesis" does not require that it be absolutely certain in an ontological sense; rather, for Barrow a true hypothesis is merely one that is consistent with the logical rules of argument and geometry, and true principles, as Malet observes, "need not be self-evidently true—they only must be free from contradiction" [1997, p. 277]. By this measure, then, principles may be drawn whether one performs one experiment or a thousand, since one is not necessarily trying to prove with absolute certainty that the principles must apply to this world, but rather only that they might.

Newton's mathematical ontology

These themes discussed by Barrow were reflected in the mathematical writings of Newton, which is not surprising given the close personal and intellectual relationship between the two men [Feingold, 1993]. In an important way, however, Newton's philosophy also transcended the debate as envisioned by Gassendi and Hobbes. There is an undeniable epistemological tension in Newton's work between his conviction that "pure" mathematics (specifically geometry) is absolutely true and certain, and his simultaneous awareness that the powers of the human intellect are limited. Again and again Newton's writings express the extreme confidence he had in his mathematical demonstrations, but he also at times backed away from making absolute claims about the reality of his mathematical demonstrations when applied to the physical world. Newton's mathematical methodology, particularly in the *Principia*, has been much discussed by historians. I.B. Cohen has described what he calls the "Newtonian style," which involves "the possibility of working out the mathematical consequences of assumptions that are related to possible physical conditions, without having to discuss the physical reality of those conditions at the earliest stages" [1980, p. 30]. This "style" relied heavily on modeling nature mathematically, but the final relationship of those models to physical reality remained a sticky issue for Newton. Michael Mahoney suggests that this issue centers on the problem of the relationship between the structure "of the mathematical model and the structure of the physical system it is meant to represent and thereby explain" [1998, pp. 749–750]. Newton wanted a genuine correspondence between mathematical models and nature, and, as Dear has suggested, one available route was mathematical constructivism. Newton's constructivism was less broadly epistemological than that of Gassendi, Hobbes, and Barrow, but it nonetheless allowed him to conceive mathematical objects as "things *to be drawn* rather than pre-existing in a Platonic realm," and thus provides an avenue for comparison of his beliefs with the goals of the nominalist authors [Dear, 1995, p. 8].[39]

Labels such as "nominalist" and "realist" break down, however, when trying to describe Newton's approach to some of the most important ontological entities in his system. Whereas realists and nominalists centered much of their debate on the ontological correspondence between concepts in the mind and idealized "objects" (such as circles or triangles) in an extramental or extrasensory realm, concepts such as Newton's "force" have no clear analog in either philosophical tradition. Force, which Newton unquestionably conceived as an ontologically real entity, is not an "object" in the same sense as either the metaphysical essence, universal category, or Platonic form of a triangle is. Neither, however, is it an artificial construction of the mind, a "concept" without a physical referent. Force is a distinctly new class of ontological being: it is real without being tangible, and its effects are mathematically quantifiable but its physical properties are unknown and perhaps unknowable. Although Newton uses geometry (and analysis) to demonstrate the existence of gravity, the force itself is not reducible to a simple geometrical object or construction, but rather to the dizzying series of geometrical demonstrations in the *Principia*. Newton also conceives force as an immanent property of God's will, and God's power, being infinite, cannot be reduced to a simple category or substance. Relating Newton's work to the concerns of the authors of the preceding decades is complicated by the fact that certain elements in his ontology—like his treatment of matter and space—do overlap with the conceptions of Gassendi, Hobbes, and Descartes, and that his mathematical epistemology (particularly the constructivist element) has parallels with Barrow's. It is possible to read a nominalist influence in Newton, because he was indeed influenced by nominalists.

We have a very good idea, from student notebooks kept during the two years before his *annus mirabilis* of 1666, that Newton's early intellectual influences were wide-ranging and diverse. Newton read Walter Charleton on atoms and Gassendist physics, Hobbes's *De Corpore*, and much of Descartes' oeuvre, as well as works by Wallis, William Oughtred, Clavius, and other important mathematical authors of his time.[40] J.E. McGuire and Martin Tamny argue for a strong atomistic interest in this early period and suggest parallels between Newton's speculation about physical atoms and his interest in Wallis' method of indivisibles. This, McGuire and Tamny propose, reveals Newton's desire to find a physical basis for the ontologically problematic mathematical entities found in the new analytic methods [McGuire and Tamny, 1983, pp. 98–99].[41] They argue furthermore that although Newton was "inclined" towards mathematical realism during this period, "with regard to the quantity of objects that exist extramentally, Newton is an Epicurean actualist." McGuire and Tamny contend that though the young Newton was very concerned with aligning mathematical representations with physical reality, he was nonetheless a "mathematical conceptualist," who recognized a divide between concepts of mathematical objects in the mind and objects in the physical world. Furthermore, they argue that this ontological concern led Newton to reject traditional infinitesimals (which he replaced with fluxions) and strongly informed the development of his mature mathematics [McGuire and Tamny, 1983, pp. 108–112].

[39] It would be an unwarranted stretch, however, to claim that Newton shared Gassendi's radical nominalistic conviction that abstract representations are always illegitimate or fictitious. Gassendi's position derives largely from his conflation of mathematics, language, and formal logic, a connection which Newton never explicitly pursued or noted.

[40] The source for this is Newton's manuscript *Quaestiones quaedam philosophicae*, which was written in 1664 or 1665. It has been edited and reprinted in McGuire and Tamny [1983].

[41] "The pattern of Newton's argument indicates that he wishes to minimize the gap between the mathematicals and the perceptible quantity of physical objects" [McGuire and Tamny, 1983, p. 98].

These questions led Newton, throughout his career, to develop a philosophy of mathematics—and particularly geometry—that was "physicalist"; that is to say, a belief that mathematical representations should be closely aligned with the properties of physical bodies and their motions. Newton's fluxional calculus differed from Leibniz's version in that Newton conceived of infinitesimal increments not as infinitely small quantities (which as objects would be difficult to justify ontologically), but rather as succeeding "moments" observed as a particle moved along a path over time.[42] The path of a curve could be treated as the combination of forces exerted on the particle as it moved, which had obvious advantages when it came to representing a body under the influence of gravity. What is important to note here is that this method of mathematical representation was actually an attempt at an accurate "description" of a physical event; the method of fluxions describes a narrative "history" (though a fictional or hypothetical one) of the motion of an object as it traverses a distance over a particular time. That this physicalist conception of mathematical representation was influenced by Hobbes's geometry is evidenced by the careful notes Newton took on *De Corpore* in his *Quaestiones*.[43]

This approach to mathematical representation also accounts for the ambivalence Newton had towards algebraic notation over the course of his career, which deepened to outright hostility as time wore on.[44] The decision to present the *Principia* in geometrical form is part of Newton's attempt to address what Niccolo Guicciardini terms "one of the most important foundational questions faced by 17th- and 18th-century mathematicians," namely, whether mathematical objects are ontologically real. In his unpublished mathematical papers, Newton compares algebra with geometry in several places. Newton states in his "Final" *Geometriae libri duo*, for example, that he does "not at all approve of the new generation of geometers" who introduce curves into analytic geometry that have not been demonstrated by traditional methods, since the curves are "arbitrary and [have] no foundation in geometry" [Newton, 1967–1981, Vol. 5, p. 471]. His concern here evidently stems from his conviction that Euclidean synthetic geometry reliably represents mechanically constructed curves, and that by accepting curves that have not been constructed in this manner, analytic geometers risk losing the rational rigor of geometry.[45] Newton

[42] As Newton explains in his final draft of the 1704 *De quadratura curvarum*, "Mathematical quantities I here consider not as consisting of least possible parts, but as described by continuous motion. Lines are described and by describing generated not through the apposition of parts but through the continuous motion of points; surface-areas are through the motions of lines, solids through the motion of surface-areas, angles through the rotation of sides, times through continuous flux, and the like in other cases. These geneses take place in the reality of physical nature and are daily witnessed in the motion of bodies" [Newton, 1967–1981, Vol. 8, p. 123].

[43] Although McGuire and Tamny note that Newton was critical of Hobbes, they also suggest that—particularly on the subject of optics—Newton was influenced by Hobbes's epistemology [McGuire and Tamny, 1983, pp. 216ff].

[44] See Guicciardini [1999, pp. 6–7]. Guicciardini argues that Newton turned from an analytical mode of expression to "a geometrical method based on limits," a move he calls "one of the most spectacular processes in the history of mathematics," comparable to Einstein's rejection of quantum theory on epistemological grounds.

[45] It should be noted here that the context for this criticism involves Newton's interest in the contemporary debate concerning the "proper" method of analysis to be used in solving problems in plane geometry. Descartes' proposal that descriptions for plane curves, for example, could be algebrized cut against the traditional geometrical analysis made famous in Pappus's *Collectio*. Newton was clearly weighing in his opinion by criticizing Cartesian analysis, and to this extent his critique is independent of the questions raised by his nominalist predecessors. One must, however, ask whether in addition to Newton's methodological commitment to the rigor of geometrical analysis (which he developed from the 1670s onward through a close reading of Pappus), he was also guided by ontological commitments. To this question we can guardedly answer in the affirmative: Newton's later adoption of geometric methods was not necessarily contingent on his earlier exposure to nominalism, but his defense of geometry was consistent with important tenets of nominalism. As Guicciardini notes, "the defense of geometry... tied in

continues by lamenting that "present day geometers indulge too much in speculation from equations," which they mistakenly interpret to provide general demonstrations. As Newton contends (adopting the traditional view), the true method for composition should be the synthetic method, and while analysis at best "guides us to the composition," "true composition is not achieved before it is freed from all its analysis." In other words, Newton is echoing Barrow's claim that reliable knowledge cannot be produced by analytical methods, since analysis is suited to the examination of specific problems, not the production of general solutions. "Simplicity in figures," Newton explains, "is dependent on the simplicity of their genesis and conception, and it is not its equation but its description whether geometrical or mechanical by which a figure is generated and rendered easy to conceive" [Newton, 1967–1981, Vol. 5, p. 477].

By defending classical methods of geometrical composition, Newton is claiming that "his method [is] more than a mere heuristic tool," as Guicciardini contends, which suggests that "geometrical interpretability guarantee[s] ontological content" [Guicciardini, 1999, p. 167]. Newton is not, however, suggesting that the geometrical representations themselves are the ontologically real entities they describe, but rather that their referents are: geometrical representations describe actual physical entities. Does this mean that Newton believes there is a one-to-one correspondence between geometrical representations and the natural phenomena they describe? He does not give a definite answer to this question. Indeed, it is clear that Newton does not expect geometry to answer metaphysical questions about nature at all; rather, Newton describes it as a method "devised, not for the purposes of bare speculation, but for workaday use," and that as a "science [it] wins our gratitude in consequence of its usefulness" (Draft of "Geometry," in Newton [1967–1981, Vol. 7, p. 289].[46] Geometry is useful because it can provide certain results, but that certainty is tied to its use of synthetic reasoning, which begins with axioms that are self-evident and which are not themselves the subject of investigation and proof.

The need for reconciliation between the deductive certainty of pure geometry and the actual existence of physical entities was necessary in order for Newton's geometry to satisfy his demand of "usefulness." This led him to a kind of constructivism, which, in Dear's words, held that "mathematical knowledge (for which, typically, geometry stands as the exemplar) is about particular kinds of objects created by active construction—not contingently, but in their very essence" (meaning that their 'essences' are constructed, not found naturally) [Dear, 1995, p. 212]. As we have already seen, one constructivist strategy was to posit that mathematical objects are merely fictions that inhabit a hypothetical world (Gassendi), or more strongly that they exist, if not in this world, then in a potential one (Barrow). Newton demanded more from his mathematical account. In the General Scholium to the *Principia*, Newton writes that "it is enough that gravity really exists and acts according to the laws we have set forth," and there is no indication that he did not conceive of gravitational force as a real entity [Newton, 1999, p. 943].[47] Newton does not indicate, however, that geometry explains what gravity is or why it exists, but rather he

with another equally important aspect of the Baconian methodology then in vogue, empiricism," which required that symbols correspond to actually existing entities [2002, p. 315]. As I have argued above, such a stance can be linked more specifically to the nominalist belief that only real singulars exist, and here Newton's exposure to Gassendi, Barrow, and Hobbes is as important as his reading of Descartes and Pappus.

[46] There is no certain date for this unpublished work, but that it had probably been written by 1693.

[47] Newton voiced a similar sentiment in a letter to Henry Oldenburg concerning mathematical optics: "I said indeed that the Science of colors was Mathematicall & as certain as any other part of Optiques; but who knows not that Optiques & many other Mathematicall Sciences depend as well on Physicall Principles as on Mathematicall Demonstrations: And the absolute certainty of a Science cannot exceed the certainty of its principles." (Newton to Oldenburg, 11 June, 1672, in Newton [1959–1977, Vol. 1, p. 187]).

holds that it describes laws (observed in the quantifiable motions of physical bodies) and demonstrates how gravity acts.

Gravity is not a "constructed" object according to Newton—it is physically real—but the objects of geometry are constructed in the mechanical sense, meaning that they are conceived as describing the motion (or combination of motions) that might be taken by a physical object in actual space. There is a link between physical description and geometrical demonstration in Newton's epistemology, but geometrical objects are not direct representations of either particular physical entities or of idealized universal forms. Attempting to describe Newton's ontology in those terms is futile: his ontology of force is literally incommensurable (in the Kuhnian sense) with the previous conceptions of both realists and nominalists [Kuhn, 1962]. This does not mean that Newton's overall natural philosophy is entirely disconnected from either Gassendi's or Descartes', but it does mean that certain concepts cannot be translated between them. What has occupied the previous philosophers in this study has been the perception, conceptualization, and representation of what I have been calling "objects." Newton's introduction of force presents a new concept, which I have called an "entity," that is not precisely synonymous with the earlier term, and thus moves the discussion of the status of mathematical ontology beyond the parameters of the medieval and early modern nominalist debate. Nonetheless, Newton's contribution to mathematical philosophy merits consideration in regard to that tradition: it is fair to say that if Newton was not strictly a nominalist (and no one has ever, to my knowledge, argued that he was), then neither was he a "realist" in the same sense as Kepler or Galileo. The nominalist influence in philosophy of mathematics certainly played a role in informing the development of Newton's early mathematical sensibility (through his readings of Gassendi, Hobbes, and Barrow), and this early influence sheds light on decisions he made in his later mathematical works.

Conclusion

What does the tradition of nominalist epistemology and philosophy of mathematics tell us about the development of mathematical natural philosophy in the 17th century? On one level, I am suggesting that nominalists like Gassendi, whose primary reputation among historians of science and philosophy has been as a "reviver" of ancient atomism, deserve greater attention as philosophers of mathematics. On a deeper level, however, I am calling for a reconsideration of major explanatory categories for 17th-century science, such as the notion of "mathematization." On the views of Gassendi, Hobbes, Barrow, and even Newton, mathematization was underwritten by a notion of construction with clear links to nominalistic and constructivist epistemology, and mathematics was regarded as a "language" for describing nature that was subject to the same epistemological conventions that govern the structure, objects, and claims to knowledge of natural languages. This view represented a departure from the competing notion that physical reality has an inherently mathematical structure, and that mathematics is an *a priori* tool for capturing the essence of that structure (as claimed, for example, by Kepler, Galileo, and Descartes). This distinction stresses the contingently descriptive, rather than ontologically necessary, relationship of mathematics to the physical world which characterized an important aspect of the nominalist epistemology, and this approach, I argue, resonated beyond the strict nominalist tradition.

As recent scholarship has noted, a debate in late sixteenth and early 17th-century mathematics provided impetus for later 17th-century arguments concerning the certainty and applicability of mathematics. The stakes in this debate, which had ties to the *Quaestio de certitudine mathematicarum* of the Renaissance, centered on whether mathematics could properly be designated an Aristotelian "demonstrative" science (that is, a science that dealt with causal knowledge). An important group of authors—particularly Italian Jesuits—contributed to this debate, and Dear and Mancosu have argued that their positions influenced the writings of Gassendi, Barrow, and Newton.[48] While my study does not question the importance of this tradition for later mathematical thought, it does further differentiate the philosophical strands that contributed to mathematical beliefs in the scientific revolution. When debating the certainty of mathematics in relation to the *Quaestio*, for example, authors such as Piccolomini and Biancani were primarily concerned with justifying the demonstrative status of mathematics in Aristotelian terms, which meant establishing the universality of mathematical demonstrations and objects. This is precisely the *opposite* of what Gassendi, for example, was attempting to do; the goal of Gassendi's nominalistic epistemology was to destroy the notion of Aristotelian universals altogether. Although it drew on older traditions, 17th-century nominalism was also distinguished from those traditions in that it did not attempt to deny the usefulness or certainty of mathematics, but rather questioned the ontological basis for all linguistic and representative mental operations. Here constructivism allowed for mathematics to make meaningful statements without necessitating a philosophy of mathematics that was ontologically realistic.

The move away from ontological realism in mathematics is another feature that has been well documented in recent years by a number of historians, and here nominalism also poses an intriguing dimension. Blay's comment that later 17th-century mathematicians, including Newton, were part of a trend pushing analytic mathematics towards "quantitative laws" that "put aside any meaningful ontological claims it might have had," is particularly intriguing. Ultimately, according to Blay, mathematics became "only a well-constructed discourse which, in no longer speaking of the reality of things, could freely employ the procedures of infinitesimal geometry and of the differential and integral calculus" [Blay, 1998, pp. 3–4, 10]. To a particular set of practitioners (including Wallis and, quite vocally, Berkeley), analytic techniques and objects were considered to be heuristic tools, a designation they obtained from an increasing identification of mathematics with language and formal logic. The rise of this kind of formalism or instrumentalism in mathematics can also be linked to the earlier nominalist tradition in mathematical philosophy promoted by Gassendi and others.[49] Although the set of conditions that produced Gassendi's initial nominalist critique of Aristotelian universals had changed drastically by the early eighteenth century, the epistemological concerns that informed mathematical philosophers like Berkeley were consistent with those of 17th-century nominalists. The next hundred-fifty years of mathematical philosophy saw an even greater increase in formalism and instrumentalism in analytic mathematics, leading to the ultimate conflation of mathematics with formal logic by Russell and Whitehead in the early 20th century [Whitehead and Russell, 1910]. Nominalism may have visibly

[48] Mancosu [1996, p. 24], for example, writes "it is thus evident from the assertions of Barrow, Gassendi, Langius, and Bayle that there is an important connection between skepticism in the seventeenth century and the *Quaestio*."

[49] There is evidence that nominalism played a significant role in Berkeley's attack on Newton's calculus of fluxions, which in turn may have prompted Colin MacLaurin's reforms in his *Treatise of Fluxions* [1742] in the mid-18th century. Jesseph [1993, p. 285] claims that far from being "a refutation of Berkeley," *Fluxions* was "a comprehensive treatise in the calculus with a Berkeleyan basis" that "essentially grants Berkeley's case against the Newtonian and Leibnizian methods and proceeds from there." See also Hill [1996, pp. 165–178].

departed from the picture by then, but its importance to the earlier tradition that sparked this trend deserves greater attention from historians of the modern mathematical sciences.[50]

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[50] As far back as 1912, Jourdain [1912, p. 624] noted that despite Georg Cantor's stated opposition to nominalism, "many men, such as Heine, Helmholtz, Kronecker, Thomae, Stoltz, and Pringsheim, have expressly advocated nominalism in mathematics and yet have sometimes made very valuable contributions even to rather fundamental questions."

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